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Estimating and testing intertemporal preferences:

A unified framework for consumption, work and savings

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ABSTRACT

Five waves of the Panel Study of Income Dynamics (PSID), 1985-1989 including both wealth supplements, are used to construct an intertemporal budget constraint for selected single headed households. A new functional form of the dual consumer profit function rationalizing consumption, labor supply and savings is specified, estimated and used to test commonly maintained separability hypotheses. Both consumption-labor and time separability are rejected. Cross-price Frisch elasticities are found not to equal zero and this in turns affects all estimates of consumption, labor supply and saving elasticities.

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INTRODUCTION

In the field of commodity demand estimation, sophisticated representations of preferences are routinely employed. With the advent of duality and flexible functional forms, parameter rich models with sufficient flexibility to uniquely identify the full set of share, income and price cross-price elasticities are used. What is minimally required in a model of n-goods are (n+4)(n-1)/2 parameters to independently identify the (n-1) shares, (n-1) income elasticities and n(n-1)/2 price cross-price elasticities. More recently in this field, Banks, Blundell and Lewbel (1997) and Ryan and Wales (1999) have derived and estimated rank-3 demand systems with a further (n-1) parameters to capture a quadratic element in expenditure. As a broad characterization of this field, it has been found that more restrictive models of demand should be rejected in favor of greater generality.

In contrast, economists who model household intertemporal choice commonly use rather crude functional forms. While data limitations, the necessity of maintaining regularity conditions, the stochastic environment or some other feature of the exercise has often confined modelers to a primal specification of preferences with tractable forms such as Cobb-Douglas, Constant Elasticities of Substitution and Linear Expenditure Systems, it can still be argued that these are early generations of models that have been rejected as being overly restrictive.

The purpose of this paper is to show the feasibility of modeling intertemporal choice in a quite general, utility consistent manner- much along the lines of demand modelers- and then test the necessity of doing so. While studies focusing jointly on consumption and labor supply quickly run into problems from the lack of data on the evolution of household consumption, what appears to be overlooked has been the availability of panel data on labor and wealth. We derive consumption expenditure quite differently using the intertemporal budget constraint. We calculate consumption expenditure by adding beginning wealth and earned and unearned income and subtracting end wealth with adjustments for rates of return on household wealth portfolios.[1] Treating dynamic optimization as a three good problem- how much to spend, how much to earn and how much to save- our specification is quite general and treats each of these choice variables in a symmetric and mutually consistent manner.

Our approach draws on what are known as Frisch demand systems or lamda (?) constant estimation (MaCurdy, 1981 and Altonji, 1986) which can be derived from a consumer profit function (Browning, Deaton and Irish, 1985). Unlike the existing literature which has been confined to simple specifications to either difference away or treat as a fixed effect the unobserved price of marginal utility, $\mu \equiv 1/\lambda$, we invert the budget constraint to determine this unobservable. This innovation both required and enabled us to apply a new functional form that has necessary and appealing properties. Our functional form is globally regular, flexible, rank 3 and provides an explicit expression of the price of utility.

We apply the profit function to test two commonly maintained, but restrictive, hypotheses; consumption-labor additivity and time separability. Avoiding the separability

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[1] Portfolio, savings and wealth are used interchangeably and are nominal quantities. We use assets to denote real wealth.

inflexibility (Blackorby, Primont and Russel, 1977) that hampers analyses based on flexible indirect utility or expenditure functions, the profit function can test these hypotheses in a straight forward manner that amounts to whether various Frisch cross-price terms enter into the system of structural equations explaining consumption, labor and wealth. Although these types of separability tests seem not to have been done in the dynamic consumer context, Barnett and Hahm (1994) among others show its application in static producer contexts. Both separability restrictions are easily rejected in favor of the most general case. This implies that Frisch cross-prices enter into the each structural equation independent of its impact of ? : wages and interest rates enter into the consumption equation; prices and interest rates enter into the labor equation; and prices and wages enter into the savings equation.

We next compare the full set of estimated elasticities of the most general model to those derived by the more restrictive models in order to evaluate the impact of these maintained assumptions. We find, for example, that the Frisch (conditional on μ) elasticity of consumption with respect to interest rates to be substantial in a general setting whereas this is constrained to zero under time separability. Because Marshallian (conditional on initial wealth) elasticity is related to a corresponding Frisch elasticity, removing the restriction affects the Marshallian elasticity of consumption with respect to interest rates. This generalization increases the estimated elasticity of consumption with respect to interest by a factor of 5 over that estimated by the restricted models. An unexpected result is the finding that interest rates, conditional on μ, have a significant impact on labor supply which is constrained to zero in restrictive models. Because homogeneity of the profit function implies certain adding up properties in the matrix of price cross-price elasticities, removing constraints on off-diagonal Frisch elasticities changes the entire set of estimated price cross-price Frisch and Marshallian elasticities.

Our procedure is able to discern how changes in wages and interest rates changes μ depending on whether this is evaluated in cross section or in time series. We argue that when the response of μ to wage increases is evaluated in cross section, this is best thought of as the effect of a transitory one year increase in wages. On the other hand, when this is evaluated in time series, this can best be though of as a perturbation to the evolutionary path of wages which has some persistence. By comparing the two, we find that the wage effect on μ in time series is over 6.5 times stronger than it is for a transitory wage increase, suggesting if perturbed wages revert to a mean geometrically, it does so at about 18% annually. Conditional on initial wealth, labor supply switches from inelastic but positively sloped in response to short term wage increases to one that is backward bending for long term wage increases due to a wealth effect.

Generalizing the modeling of household choice also changes the estimates of several important intertemporal parameters. For example, restrictive models estimate that the rate of time preference is around 5% while in our more general model it is 1.5%. Additionally, intertemporal optimization that gives rise to the Euler equation implies that the elasticity of λ with respect to interest rates should be approximately one. Our restricted models find that households are poor intertemporal optimizers with estimates of this elasticity from 0.206 to 0.232 whereas our preferred general model finds an estimate of 1.084. It appears that interest rates have two distinct channels of operation; one through the structural equations conditional on λ and another through the Euler equation representing an intertemporal optimization- but time separable models cannot distinguish the two effects.

This paper advances the modeling of household intertemporal choice in a number of profound ways. This is the first time that the profit function has been applied to the complete set of dynamic choices faced by households. We use a “hard” budget constraint- where initial wealth is predetermined or weakly exogenous. This contrasts with static analyses of consumer demand where within period total expenditure is assumed exogenous when in fact it is endogenous to the intertemporal problem. This difficulty arises in static analyses because a savings choice is not explicitly modeled which we have done here.

In one sense, we have simplified the problem. We do not think of intertemporal optimization as a maximization of utility functions that are somehow defined over vague and distant future commodity bundles. Instead, we look at the immediate choice facing households: How much do I consume, work and save today? The profit function treats each choice variable in a symmetric and mutually consistent manner. Further, our profit function is globally regular and derived from an approximation scheme with desirable properties. Most critically perhaps, we show that models with separability restrictions which inform most of our current understanding of labor, consumption, savings and intertemporal elasticities should be rejected in favor of a general model such as ours when modeling household intertemporal choice.

The following section presents a theoretical discussion of our approach and the new functional form. This is followed by a section describing the data used for this analysis. This is followed by a discussion of our results and a conclusion where we emphasizes the many possible extensions to this basic framework that we hope economists find fruitful.

ECONOMIC MODEL

In this section, we present the intertemporal decision as a one year problem where households evaluate prices, wages and interest rates together with an initial level of wealth to plan their consumption, labor and savings choices. We develop the connection between a household's present cost of future consumption and a contemporaneous interest rate and introduce a nominal price of next period consumption which we call an interest factor. We discuss the profit function which incorporates our interest factor as well as theoretical features of our model. As will be seen in the next section, the data available does not include information on beginning-of-year and end-of-year wealth but rather wealth at the beginning and end of a five-year span. The issue of matching this model with 5 year's of incomplete data is addressed in the data section but it suffices here to state that this is considered as a sequence of 5 one-year optimizations with household re-optimizing with the realization of new wage and interest information each year.

Consider a common specification of the intertemporal budget constraint: $W_t + y_t + w_t h_t - p_t c_t - (1+i_t)^{-1} W_{t+1} = 0$ where W is the nominal value of wealth, y is tax adjusted unearned income, w is after-tax nominal wage, h is hours of work, p is consumer prices, c is consumption, i is nominal interest and subscript t denotes time. The choice variables are c_t, h_t and W_{t+1}. We have mixed real and nominal variables in this specification. Define the real wealth variable $A_t = W_t/p_t$. The budget constraint can now be written terms of real choice variables: $W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1} = 0$ where $r_t = p_{t+1}/(1+i_t)$ is the present nominal price of future real consumption and wealth. We call r_t the time t interest factor because dividing this by p_t gives a *real* time t cost of next period consumption. Dividing the LHS of the intertemporal budget constraint by p_t expresses the budget constraint exclusively in terms of real prices and quantities. The constraint, written in this form, alerts us to the fact that the three real choice variables; c_t, h_t and A_{t+1} are linear in prices p_t, wages w_t, and interest factor r_t and that predetermined nominal wealth W_t and exogenous income y_t has the effect of shifting the level at which the constraint binds. Clearly then, p_t, w_t and r_t are the correct prices corresponding to consumption, labor supply and future real wealth.

Intertemporal maximization, what is called the primal problem, is often modeled with a recursive value function.[2] Consider the value function

$$
\begin{aligned}
&V(W_t + y_t, p_t, w_t, r_t : c_{t-1}, h_{t-1}) = \\
&\max_{c_t, h_t, A_{t+1}} \left[U(c_t, c_{t-1}, h_t, h_{t-1}) + E_t \delta V(p_{t+1} A_{t+1} + y_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) \right] \qquad (1) \\
&\text{s.t.} \quad W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1} = 0
\end{aligned}
$$

where V is the value function, U is the utility function, E is the expectations operator with respect to future dated information and δ is the time discount factor. The appearance of past consumption in the utility function allows for durability in the case of $U_{c(t)c(t-1)} < 0$ and for habits in the case of $U_{c(t)c(t-1)} > 0$.[3] A symmetric time dependence is allowed for in work hours allowing the disutility of work to change with prior work experience. In the case of durability or habit formation, the utility function is not time-separable and present consumption and work will affect future utility.

The Lagrangian for the primal problem is

$$\begin{aligned} L &= U(c_t, c_{t-1}, h_t, h_{t-1}) + E_t \delta V(p_t A_{t+1} + y_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) + \\ &\lambda_t (W_t + y_t + w_t h_t - p_t c_t - r_t A_{t+1}) \end{aligned} \tag{2}$$

with first order conditions over present choice variables,

$$\begin{aligned} L_c &= U_c + E_t \delta V_c - \lambda p = 0 \\ L_h &= U_h + E_t \delta V_h + \lambda w = 0 \\ L_A &= E_t \delta V_A - \lambda r = 0 \end{aligned} \tag{3}$$

where time subscripts are removed for now.

The total differentiation of the 3 first order conditions (3) with respect to 3 prices; p_t, w_t and r_t gives the 9 equations,[4]

$$\begin{bmatrix} L_{cc} & L_{ch} & L_{cA} \\ L_{ch} & L_{hh} & L_{hA} \\ L_{cA} & L_{hA} & L_{AA} \end{bmatrix} \begin{bmatrix} c_p & c_w & c_r \\ h_p & h_w & h_r \\ A_p & A_w & A_r \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \tag{4}$$

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[2] Meghir and Weber (1996) for example.
[3] Throughout this paper, the subscripting of a function with an argument shall denote differentiation with respect to the argument. Time and indexing variables may be denoted by subscripts or by parentheses.
[4] By symmetry, 6 of the 9 equations are independent.

Let [A] and [B] denote respectively the first and second matrix on the LHS. In the most restrictive case we consider where utility, $U = U^c(c_t) + U^h(h_t)$, is additive, the value function does not have prior consumption or work hours. Additionally, the cross derivative $U_{ch} = 0$. This means that [A] is a diagonal matrix which implies [B] is also diagonal. The zeros in the off-diagonal elements of [B] implies that the structural equations for consumption, work and wealth are functions of own price only. The inversion of the first order conditions (3) give the consumption function, $c_t^* = f^A(\lambda_t p_t)$, the labor supply function, $h_t^* = g^A(\lambda_t w_t)$, and the wealth function, $W_{t+1}^* = h(\lambda_t r_t)$[5] where the asterisk denotes model predicted quantities.

Relaxing additivity, the time separable utility function $U = U^{TS}(c_t, h_t)$ has first order conditions $U_c^{TS}(c_t, h_t) = \lambda_t p_t$ and $U_h^{TS}(c_t, h_t) = -\lambda_t w_t$. This adds a non-zero element L_{ch} to matrix [A] and implies a corresponding non-zero elements in matrix [B]. Inversion now leads to the consumption function $c_t^* = f^{TS}(\lambda_t p_t, \lambda_t w_t)$ and the labor supply function $h_t^* = g^{TS}(\lambda_t p_t, \lambda_t w_t)$. Wages now appear in the consumption function and prices appear in the labor supply function.

In the most general case where consumption-labor additivity and time separability are relaxed, the appearance of prior consumption and labor supply in the value function implies that none of the off-diagonal elements of [A] are zero which implies none of the off-diagonal elements of [B] are zero. Thus our most general case will have a system of

[5] We have ignored the mechanism whereby w_t and r_t udates expectations. This is modeled later in the paper.

structural equations $c_t^* = f^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t)$, $h_t^* = g^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t)$ and $W_{t+1}^* = h^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t,)$ where all prices enter into each equation. Our objective then is to develop a utility consistent system of equations for consumption, labor and savings and examine the significance of these cross-price terms.

We use duality theory (Diewert, 1974) rather than the specification of a particular utility function in a primal approach to identify preferences. Duality techniques specify a parent function that spawns the structural equations via differentiation with respect to a choice variable's price. This is a considerable convenience and circumvents the problem of inverting the first order conditions (3) to find a solution. The consumption, labor supply and wealth functions are obtained simply by taking the derivative of the profit function with respect to own price. Thus for example, estimated consumption, $c^* = -p_p$ where the subscript denotes partial differentiation and estimated consumption expenditures $pc^* = -pp_p$.[6] Properties required of the consumer profit function are homogeneity and convexity, necessary conditions given a well defined maximization problem.[7] There are other advantages of the dual approach related to the modeling of the evolution of λ which we discuss more fully in the data section.

A desirable property of any dual functional form is flexibility. Consider any arbitrary utility function evaluated at a certain point in n-goods space. The first derivative of the utility function at this point lead to n gradients and the matrix of second derivatives

[6] Note that consumption and assets are given by the negative of the partial derivative while labor hours are given by the positive.

of the utility function contained in the Hessian lead to curvatures in n(n+1)/2 directions. Since an affine transformation of the utility function is innocuous because it is an equally valid representation of preferences, what is material is the (n-1) relative gradients and the (n(n+1)/2 -1) relative curvatures. A functional form with sufficient parameters to independently estimate each of these relative gradients and curvatures of an arbitrary utility function is said to be flexible.[8]

The dual profit function associated with the value function (1) is

$$\pi(p_t, w_t, r_t, \mu_t) = \max_{c_t, h_t, A_{t+1}} \begin{bmatrix} U(c_t, c_{t-1}, h_t, h_{t-1}) + E_t \delta V(W_{t+1} + y_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) \\ + \lambda_t (w_t h_t - p_t c_t - r_t A_{t+1}) \end{bmatrix} \tag{5}$$

where $\mu_t \equiv 1/\lambda_t$ is the inverse of the time-t Lagrangian multiplier of the constraint or the price of marginal utility. Recognizing data limitations that lie ahead, prior consumption and work and future prices are subsumed in the profit function in accordance with our general approach of determining current choice variables from available exogenous variables. The first order conditions of the profit function (5) are *identical* to the first order conditions of the Lagrangian (3) of the primal value function.

An important feature of the dual profit function (5) is the role played by λ. The profit function conditions on λ and is silent on its determination. Note that the expectation is over the arguments of the following period's value function; namely exogenous income, y, prices, p, wages, w, and interest rates, r, from time t+1 and into the

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[7] Reference to theoretical contributions to the consumer profit function since Deaton, Browning and Irish (1985) can be found in Kim (1993), Chaudhuri (1995, 1996) and McLaughlin (1995).

future. The term λ is not involved with the expected value function for the following period but rather multiplies a linear combination of time t prices and time t choice variables. It is this multiplicative feature that gives us the convenient property whereby the derivative of the profit function gives us the corresponding choice variable. Clearly, additional structure is needed for the empirical implementation of the profit function. The term λ has to be supplied independently to the profit function and resulting structural equations for which we will use the ex-post budget constraint and assumptions about its evolution.

One of the challenges of taking a new approach to the data is often developing a parametric specification to fully rationalize the data. In addition to the homogeneity and convexity of the profit function in the observable price variables, consideration needs to be given for the unobserved μ. We develop a model that is not only globally convex in prices and in μ, it lends itself to an explicit expression of μ on inversion of the dynamic budget constraint. As far as we are aware, this is a new functional form. Our profit function is parameterized by

$$\pi(p, w, r, \mu : z) = A(P, \mu, \alpha)\mu + B(P, \beta : z) + C(P, \gamma) / \mu \qquad (5)$$

where P = (p_1, p_2, p_3)' = (p, w, r)' is a vector of prices, z=(age, age^2, age^3, number of dependents, sex of household head)' is a vector of demographic characteristics[9] and α, β and γ are vectors of parameters to be estimated. The sub-functions $A(P, \mu, \alpha)$,

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[8] Flexible in our paper does not mean minimally flexible, another usage, which implies the minimum number of parameters required for flexibility.

$B(P,\beta : z)$ and $C(P,\gamma)$ are given by

$$A(P,\mu,\alpha) = \sum_{i=1}^{3} \alpha_{ii} \ln(\mu / p_i) + \sum_{i=1}^{3} \sum_{j>i} \alpha_{ij} \ln(\mu /(p_i + \alpha_{ijj} p_j)) \tag{6}$$

$$B(P,\beta : z) = \\ \sum_{i=1}^{2} d_i [\beta_{i10} p_1 + \sum_{j=2}^{3} p_j (\beta_{ij0} + \beta_{ij1} age + \beta_{ij2} age^2 + \beta_{ij3} age^3 + \beta_{ij4} dependents)] \tag{7}$$

where d_i is the indicator for the sex of the household head, i=1 indicating male and i=2 indicating female[10], and

$$C(P,\gamma) = [(\gamma_{11} p + \gamma_{12} w + \gamma_{13} r)^2 + (\gamma_{22} w + \gamma_{23} r)^2 + (\gamma_{33} r)^2] / 2 \tag{8}$$

It can be readily verified the profit function is homogeneous and that $\alpha_{ij} > 0$ and $\alpha_{ijj} > 0$ is sufficient for global convexity.

To illustrate some of the properties of this model, consider now the structural equation for end wealth of a male head of household. Our wealth equation is found by differentiating the parent profit function with respect to r to give

$$- A^* = \partial\pi / \partial r = A_r (P,\mu,\alpha)\mu + B_r (P,\beta : z) + C_r (P,\gamma) / \mu \tag{9}$$

with differentiated sub-functions

$$A_r (P,\mu,\alpha) = -\left(\frac{\alpha_{13}\alpha_{133}}{p + \alpha_{133} r} + \frac{\alpha_{23}\alpha_{233}}{w + \alpha_{233} r} + \frac{\alpha_{33}}{r} \right)$$

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[9] The age variable is reported age of head minus 45 years in order to center the regression around prime aged heads.

[10] Additional terms with β parameters of the form $\beta_{ij} p_i^{0.5} p_j^{0.5}$ with convexity restriction $\beta_{ij} \le 0$ were also tried in the regression. These constraints were binding in every regression performed.

$$B_r(P,\beta : z) = \beta_{130} + \beta_{131}\text{age} + \beta_{132}\text{age}^2 + \beta_{133}\text{age}^3 + \beta_{134}\text{dependents} \quad \text{and}$$

$$C_r(P,\gamma) = \left((\gamma_{11}p + \gamma_{12}w + \gamma_{13}r)\gamma_{13} + (\gamma_{22}w + \gamma_{23}r)\gamma_{23} + \gamma_{33}^2 r\right).$$

A total of 16 parameters determine the wealth equation of male headed households. By differentiating the profit function with respect to wages and prices, it is easily verified that 15 parameters determine the labor supply equation and 9 parameters determine the consumption equation.

Each structural equation such as (9) conditions multiplicatively on μ and $1/\mu$. Together with the derivative of the sub-function B which gives the third function, this Frisch system might be called rank 3 (Lewbel, 1991) drawing obvious analogies with indirect utility functions where μ replaces income. Additionally as μ and $1/\mu$ enters the structural equations, this can be considered as a first order Laurent approximation in μ. As demonstrated theoretically by Barnett (1983), the Laurent series approximation has superior fit compared to a Taylor series approximation of the same order and subsequently led to the Miniflex family of demand systems.

Our profit function has considerable generality which we highlight be drawing analogies to flexible functional forms. The unrestricted off-diagonal parameters γ_{12}, γ_{13} and γ_{23} identify the off-diagonal cross-price responses of matrix [B] in (4). The unrestricted diagonal γ_{22} and γ_{33} parameters identify how the structural equations change with respect to changes in $1/\mu$ and parallel the parameters that identifies income responses in indirect utility systems.[11] The unrestricted β parameters identify levels with sufficient parameters for flexibility and additionally capture suspected demographic and lifecycle influences. The diagonal parameters α_{22} and α_{33} are analogous to the third rank in rank 3 systems[12] while the off-diagonal parameters α_{12}, α_{13} and α_{23} identify how the structural equations change with respect to cross-price terms conditional on μ for additional generality.

The economic restrictions of consumption-labor additivity and time separability are now easily cast as simple parametric restrictions on our structural equations which we describe now in order of increasing generality. The first of these we call the basic regression, our most parsimonious case. This regression sets all cross terms $\alpha_{ij} = 0$ and $\gamma_{ij} = 0$ for $i \neq j$ and is implied by consumption-leisure additivity and time separability. This sets all off-diagonal elements of matrix [B] in equation (4) to zero. Additionally, all β parameters except 6 β_{ij0}, i=1, 2 and j=1, 2, 3, are set to zero removing the impact of age and number of dependents from the structural equations.

The first generalization allows demographic variation to impact the levels c, h, and A. One expects labor supply and wealth demand to follow lifecycle patterns and this is accomplished by estimating an additional 16 β parameters associated with age, age squared and age cubed and the number of dependent. We call this case the demographic regression.

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[11] We sometimes estimate γ_{11} while at other times hold this as a constant. When estimated, we have additional generality. This is discussed later.
[12] We do not estimate α_{11} for reasons discussed later.

The second generalization allows for consumption-leisure non-additivity. This is accomplished by allowing parameters α_{12} and γ_{12} to take on non-zero values. This case allows for the identification of the element c_w (and by symmetry h_p) in matrix [B] in equation 4 and if non-zero implies that element L_{ch} in matrix [A] is non-zero. This case we call the time-separable regression. The third generalization allows for intertemporal non-separability. This is accomplished by allowing the remaining parameters α_{13}, α_{23}, γ_{13} and γ_{23} to take non-zero values. This of course fills the remaining elements of matrix [B] and, if non-zero, implies matrix [A] has non zero elements. We call this case the general regression.

We now discuss the determination of unobserved μ. A similar approach has been adopted by Cooper, McLaren and Wong (2001) to a static representative consumer problem and McLaren, Rossitter, and Powell (2000) to determine unobserved utility in an expenditure function in a static macroeconomic setting. If beginning and ending wealth, exogenous income, wages and interest rates are known, the unobserved μ is then implicitly defined by the budget constraint $W + y + p\pi_p + w\pi_w + r\pi_r = 0$. Expressing this more fully by explicitly showing the differentiation of the each of the sub-functions to the profit function, we have

$$
\begin{array}{l}
W + y + \\
pA_p\mu \quad + \quad pB_p \quad + \quad pC_p/\mu \quad + \\
wA_w\mu \quad + \quad wB_w \quad + \quad wC_w/\mu \quad + \\
rA_r\mu \quad + \quad rB_r \quad + \quad rC_r/\mu \quad = \quad 0
\end{array} \tag{10}
$$

Multiplying (10) by μ provides a quadratic formula in μ. Similarly, multiplying (10) by λ provides a quadratic formula in λ. To collect like terms in (10) and for notational convenience, let

$$A^* = \sum_{i=1}^{3} p_i A_{p(i)} = (-\alpha_{11} - \alpha_{12} - \alpha_{13} - \alpha_{22} - \alpha_{23} - \alpha_{33})$$

$$B^* = W + y + \sum_{i=1}^{3} p_i B_{p(i)} = W + y + B(P, \beta : z)$$

$$C^* = \sum_{i=1}^{3} p_i C_{p(i)} = 2C(P, \gamma) .$$

The second equality for A* is easily verified and the second equality for B* and C* hold because of homogeneity of degree 1 and 2 respectively. The positive roots of quadratic equation (10) expressed in these alternative forms are, respectively,

$$\mu = \frac{-B^* - \sqrt{B^{*2} - 4A^*C^*}}{2A^*} \tag{11}$$

and

$$\lambda \equiv 1/\mu = \frac{-B^* + \sqrt{B^{*2} - 4A^*C^*}}{2C^*} . \tag{12}$$

Quite serendipitously, restrictions sufficient for convexity, $\alpha_{ij} \geq 0, \alpha_{ijj} \geq 0$, lead to $A^* \leq 0$ and together with the quadratic form $C^* \geq 0$ becomes sufficient for a globally positive discriminant and a real root. These expressions for μ and λ, which are now solely in terms of observable variables, are substituted into the structural equations such as (9).

The sub-functions A, B and C have an economic interpretation that is noteworthy. Basically, the sub-function A captures asymptotically the preferences of the infinitely rich which we define as those with μ approaching infinity. The sub-function C captures asymptotically the preferences of the infinitely poor which we define as those with λ approaching infinity. The sub-function B occupies a middle ground and acts as an intercept and pivot point around which conditional-on-μ and conditional-on-λ demands radiate.

To illustrate the preceding, we use the basic regression with the minimum of parameters for a 45 year old male head with no dependents. The structural equations for real consumption, labor supply and assets are respectively,

$$\begin{aligned} -c &= -\alpha_{11}\mu / p + \beta_{110} + \gamma_{11} p\lambda \\ h &= -\alpha_{22}\mu / w + \beta_{120} + \gamma_{22} w\lambda \\ -A &= -\alpha_{33}\mu / r + \beta_{130} + \gamma_{33} r\lambda \end{aligned} \tag{13}$$

or structural equations for consumption expenditure, labor earnings and wealth,

$$\begin{aligned} -pc &= -\alpha_{11}\mu + \beta_{110} p + \gamma_{11} p^2\lambda \\ wh &= -\alpha_{22}\mu + \beta_{120} w + \gamma_{22} w^2\lambda \\ -rA &= -\alpha_{33}\mu + \beta_{130} r + \gamma_{33} r^2\lambda \end{aligned} \tag{13'}$$

Now consider the numerator of equation (11), $-B^* - \sqrt{B^{*2} - 4A^*C^*}$. As B* approaches positive infinity, the term -A*C* becomes inconsequential in the discriminant and the numerator approaches -2B*. On the other hand, as B* approaches negative infinity, the numerator approaches -B*-|B*| = 0. The numerator of equation (12),

$-B^* + \sqrt{B^{*2} - 4A^*C^*}$, acts in the opposite manner approaching 0 as B* approaches positive infinity and converges to -2B* as B* approaches negative infinity.

Finally, consider the denominator of equation (11). This is equal to $2(\alpha_{11} + \alpha_{22} + \alpha_{33})$ in the basic regression. Substituting (11) into (13'), it is now obvious that as B* approaches infinity, marginal increases in wealth are apportioned to consumption expenditure, the absence of work earnings (because of the negative sign) and end-of-period wealth in the ratio $\alpha_{11}/(\alpha_{11} + \alpha_{22} + \alpha_{33})$, $\alpha_{22}/(\alpha_{11} + \alpha_{22} + \alpha_{33})$ and, $\alpha_{33}/(\alpha_{11} + \alpha_{22} + \alpha_{33})$ respectively. A similar operation occurs with the sub-function C which determines how marginal increases in wealth is apportioned to the three choice variables as B* approaches negative infinity and is determined by the γ parameters and by prices. This apportionment determined by the parameters of sub-function A we shall call the "A-effect," and the apportionment determined by the parameters of sub-function C we shall call the "C-effect."

In estimation, the set of β parameters act as intercept parameters identifying the central location of the cross-sectional distribution of consumption, labor or wealth. Since one expects demographic and life cycle factors to have an important influence on preferences, a total of 22 β_{ijk} parameters are added to distinguish household heads by gender, age and number of dependents in the system of structural equations and their significance evaluated empirically.[13] We shall call both the pivotal nature of intercept

[13] Age and dependents parameters were not added to the consumption equation. The reason for this is discussed in the next section.

identification and the impact of demographics identified by parameters in sub-function B the "B-effect."

How the structural equations are non-linear in μ is an important attribute of this model and solved a particularly challenging problem encountered with what might be called linear-in-μ or linear-in-λ rank 2 models. Consider the case where C(P:γ)=0 and the structural equations are linear in μ. We found that the β parameters again estimated the location of the data but derived values of μ took positive and negative values as the regression pivots linearly around the central location of the distribution. Negative values for the price of utility violate regularity and any economic sense. Negative values were found in approximately half the sample irrespective of whether the sub-functions C or A were identically zero.

As mentioned above, beginning and end wealth are not available for just one year as has been supposed here- but fortunately, this is not a major complication. We turn next to a description of the data and how this was made operational despite data shortcomings.

DATA

The data for this study comes from the Panel Study of Income Dynamics (PSID), a continuing study started in 1968 with approximately 4,800 households. Since that time, the panel has grown as numbers of new household formed have exceeded those that have attrited (Hill, 1992). Five waves of the family files from surveys fielded from 1985 to 1989 were used which each contained detailed income and work data for the previous year. Additionally, these files include two wealth supplements that asked about wealth in the beginning and end of this span. By the time of interview, many households have had long standing participation with this survey.

To build a balanced panel across 5 years, single headed households who remained heads from 1984 to 1989 were selected. This choice does not allow for changes in what the PSID calls major adults but allows other changes to the households such as the birth or adoption of children or children leaving home to establish one of their own.

We removed all cases where income, wage or wealth was top-coded in any year by the PSID.

Next, a measure of real return on wealth for each year is calculated. We treat wealth as a single amorphous good (as the literature commonly does for consumption and labor supply) and add all disparate returns to wealth into a single total. We derive real returns by adding all recorded receipts from wealth: net of tax income received from financial assets such as rent, dividends and interests; and the asset portion of income from unincorporated businesses, farming, market gardening and roomers and boarders as determined by the PSID and net out the impact of marginal tax rates on this income. To this taxable total, one fifth of the 5-year capital gains as determined by the PSID is added without tax on the assumption that the capital gain is unrealized and therefore not taxable. Finally, we add an imputed rental services on owner occupied housing. We treat the decision to purchase a home as an investment, rather than consumption, decision and therefore add its tax-free benefit to a general total to compute the return on wealth.

This total return on wealth was divided by an interpolated level of wealth based on net wealth in 1984 and in 1989. We use the variable net wealth as defined by the PSID which includes the main home, other real estate, farms or businesses, stocks, cash accounts and other items, but exclude the value of motor vehicles.

The income return on wealth, i_t, is given by

$$i_t = \frac{(1 - mt_t)\text{total asset income}_t + 0.05\text{house value}_t + \text{capital gain} / 5}{(W_t + W_{t+1})/2} \qquad (14)$$

with $W_t = W_{1984} + (t\text{-}1984)(W_{1989}\text{-}W_{1984})/5$ and mt_t equal to time-t marginal federal income tax rate.

In the money demand literature, for example Barnett, Fisher and Serletis (1992), expressions are sought for the return on different kinds of money assets and a household's return is based on their mix of these assets. This assumption is appropriate if the monetary asset classes are homogeneous and there is a competitive market which brings about one price (or return) for each asset class.

Here, we use the household's income and capital gain on their assets to derive an individual specific return on assets rather than infer a general rate based on an average market rate of return on financial assets, say the fixed term bond market. Cross-sectional variation in wages in the labor supply literature is often explained by differential productivities of workers such that firms are paying a fixed wage rate per unit of productivity. In a parallel fashion, we consider it reasonable that households may also have different productivities in obtaining yields on their assets. The return on assets we consider- all disparate items that make up wealth- is much more heterogeneous than that based narrowly on just monetary or financial assets. For instance, a proprietor of a business is likely to have intimate knowledge of their unique return on a particular investment in their business which will not be arbitraged because of the absence of markets. Our analysis is interested in the range of returns felt by households on the entirety of their wealth and their response to their idiosyncratic return rather than their response to a return available in any specific market.

All households with starting or ending wealth of less that $500 were deleted. Some of these cases showed implausibly large values for i_t which is understandable given the denominator of (14) is small. Additionally, all households with i_t greater than 40% or less than -20% in any of the five years were also deleted.

Next, a value for 5 year consumption is calculated for each household. This is calculated as the residual from the ex-post budget constraint. The budget constraint for a one year period is $W_t + y + w_t h_t - p_t c_t - r_t A_{t+1} = 0$. By recursive substitution, the 5 year budget constraint is

$$W_{1984} + \sum_{t=1984}^{1988} rr_t (y_t + w_t h_t - p_t c_t) - rr_{1988} r_{1988} A_{1989} = 0 \tag{15}$$

where $rr_{1984}=1$ and $rr_j = \prod_{t=1985}^{j} r_t / p_{t+1}$, j=1985,…1988. While it is not possible to calculate consumption in each period, we can calculate 5 year composite consumption by

$$\sum_{t=1984}^{1988} rr_t p_t c_t = W_{1984} + \sum_{t=1984}^{1988} rr_t (y_t + w_t h_t) - rr_{1988} r_{1988} A_{1989} \tag{16}$$

where all terms on the RHS are observable and based on ex post prices.

Using this measure of consumption, several cases of negative consumption expenditures were observed. For these cases, final assets were too high given beginning assets and recorded incomes. These cases were deleted as were cases which had calculated 5 year consumption expenditures less than $5,000 which we arbitrary define as a subsistence level of expenditure.

The term y_t which measures time t exogenous income was calculated as income from all public and private transfers plus inheritances plus federal marginal tax rates times pretax labor earnings less federal income taxes. Since we wish to capture household decisions at the margins, it is appropriate to use $(1\text{-}mt_t)$pretax wage$_t$ as the real benefit of working the marginal hour in year t. However, since we have a progressive income tax system, marginal tax rates multiplied by gross labor earnings overstates the amount of tax paid on labor earnings. As the objective of the consumption equation (16) is to calculate the present value of consumption from observables, we add back marginal tax multiplied by earning and subtract federal income taxes since this information is available. This has the effect of adding what many economist call virtual income.

After removing observations as described, 525 observations were left. Summary statistics for both male and female headed households are recorded in the table 1.

[TABLE 1 ABOUT HERE]

The model counterpart to the ex-post 5-year budget constraint (15) is accomplished by supplying the corresponding Frisch demand for each quantity. Naturally, we will supply the prices that prevailed at the time the choice was made. In each period, household exercised 3 choices: a consumption, work and savings decision, maximizing their value function (1). We wish to impose the minimum structure necessary to estimate parameters of the model. So, although households made 5 endogenous A_t choices for t= 1985,…,1989, since only the final year is available in the data, we do not impose any structure on the earlier unknown wealth choices. Similarly, the household made 5 endogenous c_t choices for t= 1985,…,1989 which cannot be determined individually. We impose no structure on the 5 year ex-post budget (15), other than the appropriate time t Frisch demand represented the choices made. Thus, the model counterpart to (15) is

$$W_{1984} + \sum_{t=1984}^{1988} rr_t (y_t + w_t \pi_w(t) + p_t \pi_p(t)) + rr_{1988} r_{1988} \pi_r(1988) = 0 \qquad (17)$$

It can be seen from profit function (5), which in turn is derived from value function (1), that Frisch demands represent a household's best effort at intertemporal optimization given time t information. It is certainly possible for a household to regret a decision made previously given the realization of new information. It is also true that a household will evaluate future prices as it affects the expected next period value function in making there present choices but that this choice is made in light of present information only.

The model counterpart of the budget constraint (17) reveals another advantage of the dual approach we have used. In the dynamic context, consumption and work are control variables while asset is a state variable. Ending asset is an endogenous variable if we are to model this choice; however it becomes a predetermined, weakly exogenous variable in the following period. When a primal approach is used for the specification of preferences, one must employ recursive substitution schemes but this becomes intractable when there is any generality to the utility function. The dual approach we employ adds an additional state variable, λ, which we can model independently.

Our approach gives rise to 7 structural equations that can be matched with the data: one for 1989 wealth, one for 5 year composite consumption and 5 for labor supply in each of the years 1984 to1988. However, consumption was not independently determined but rather imputed from (16), a function of 1989 wealth and 5 year's labor supply. Similarly, the model counterpart to 5 year composite consumption is required to implicitly define μ. Thus, we use the 6 independent estimating equations;[14]

$$w_t h_t = w_t \pi_w(t) + \varepsilon_t \qquad \text{for } t=1984,\ldots 1988, \text{ and}$$

$$r_{1989} W_{1989} = -r_{1989}\pi_r(1988) + \varepsilon_{1989} \tag{18}$$

appending ε_t as period t error. We assume that $\varepsilon = (\varepsilon_{1984}, \varepsilon_{1985}, \varepsilon_{1986}, \varepsilon_{1987}, \varepsilon_{1988}, \varepsilon_{1989})'$ is multivariate normal and we estimated parameters using the full information maximum likelihood procedure implemented in TSP version 4.5.

We multiply the wealth equation by r_{1988} and each period t labor supply equation by w_t, a practice common in the demand analysis field that allows for adding up in the budget constraint. For our purpose, it also has the effect of removing households with non-working heads from the regression.[15] Thus we use all the 525 observations available to estimate our parameters while only those working in any year contribute to the

[14] A consumption expenditure estimating equation could be added to the list but the error from this equation is not independent of the other 6 errors as the sum of all 7 errors is identically zero.

estimation of parameters associated with wages and labor supply in the year they worked. While this is only a subset of the sample (see table 1), we have 5 years of repeated measures recording their work choices.

As the 6 estimating equations (18) stand, they contain 5 unobserved arguments, μ_t while the budget constraint (17) allows us to solve only one additional variable. Our strategy is to use the budget constraint to determine an individual specific μ_i and offer readers a menu of 3 choices which relate μ_{it} to μ_i where we introduce the i indexing subscript to denote household i. These choices will be guided by what readers feel are reasonable behavioral assumptions about the ability of households to intertemporally optimize. We are not partisan to any specific formulation on the evolution of μ and consider it simply an empirical matter.

Each of these alternatives will have the form $\mu_{it} = f(p, w_i, r_i, y_i, W_i : \delta, t)\mu_i$ where p, w, r, y, and W denote a 5 year vector containing the corresponding price, wage, interest factor, exogenous income or wealth series, δ denotes a time discount factor and the subscript denotes household i, and can be substituted into equation (17). Function f is required to be homogeneous of degree zero in prices, exogenous income and nominal wealth. Further, function f in its most general form exhausts all price and wealth information available in our analysis. The implicit definition of μ in equation (17) makes

[15] Our purpose is not to explain the dichotomous decision to work or not for which a reservation wage needs to be constructed.

this, like function f, a function of nearly all available information in the general regression. We show, however, that these can be made to play different roles.

The first of our models supplying the 5 required functions we call the fixed effects model and corresponds to $\mu_{i,t} = (p_t / p_{1988})\mu_i$. The fixed effects model holds the real price of utility constant for each household i for the 5 time periods t. Consequently, the is no time-series variation of $\mu_{i,t}$ within a household. Cross-sectional differences in beginning wealth, exogenous income, wages and interest rates are the sole factor creating cross-sectional differences in the real price of utility in this specification. The second case we offer we call the perfect foresight model and is defined by $\mu_{i,1988} = \mu_i$ and $\mu_{i,t} = p_t / p_{1988} \prod_{j=t}^{1987} \delta r_{i,j} / p_j \mu_i$. This specification assumes the perfect foresight Euler equation is operational where time discount parameter, δ, is an additional parameter requiring estimation. In this specification, cross-sectional variation drive differences in the price of marginal utility for the year 1988,[16] however, household specific time t price of marginal utility for other years are driven by household specific interest returns. The assumptions justifying the perfect foresight Euler equation are intertemporal optimization and certainty. The third case we offer we call the stochastic model where $\mu_{i,t}$ is perturbed around μ_i by realizations of household specific real time t variables. The stochastic model specifies

$$\mu_{i,t} = \frac{\delta^{(1988-t)} p_t}{p_{1988}} \exp(\theta_w (\tilde{w}_{i,t} - \tilde{w}_i) + \theta_r (\tilde{r}_{i,t} - \tilde{r}_i) + \theta_y (\tilde{y}_{i,t} - \tilde{y}_i) + \theta_a (A_{i,t} - A_i))\mu_i \quad (19)$$

where δ, θ_w, θ_r, θ_y and θ_a are 5 additional parameters requiring estimation and tilde ~ denotes the real wage, $w_{i,t}/p_t$, real interest factor, $r_{i,t}/p_t$, or real exogenous income, $y_{i,t}/p_t$. The term $\tilde{w}_i = \sum_{t=1984}^{1988} \tilde{w}_{i,t}/5$ is household i's 5 year real average wage. We interpret this as an estimate of a household's "permanent wage" reflecting all relevant fixed effects, for example the education and qualifications of the household head. Household real interest return, exogenous income and assets are defined and interpreted similarly. This stochastic specification allows for time discounting of real marginal utility. The exponent incorporates shift parameters θ when real wages, real interest factors, exogenous income or assets are perturbed in time t from a household's 5 year average.

RESULTS

This section reports on some of the challenges of estimation, how these were overcome and results obtained. As is common in non-linear estimation, functions of parameters were often easier to identify that the parameters individually. We found that the remaining α_{ij} and γ_{ij} parameters were far easier to estimate once one non-zero α and one non-zero γ were specified as constants. This is understandable if one considers the structural equations. Consider the (negative) wealth expenditure equation (9) reproduced below,

$$-rW^* = r\pi_r = rA_r(P)\mu + rB_r(P;z) + rC_r(P)/\mu$$

[16] The fit of the regressions are unaffected by which year is chosen.

and the explicit function for μ,

$$\mu = (-B* - \sqrt{B*^2 - 4A*C*})/2A*\ .$$

It can be seen on substitution that the wealth equation changes with the term $-B* - \sqrt{B*^2 - 4A*C*}$ in the proportion $rA_r/2A*$ which is simply a ratio of α parameters. The same is also true of the γ parameters where the wealth equation changes with the term $-B* + \sqrt{B*^2 - 4A*C*}$ in the proportion $rC_r/2C*$. Except for the appearance of the α and γ parameters in the discriminant, the structural equations would be homogeneous of degree zero in these parameters. As it was not possible to estimate all α and γ parameters simultaneously, we arbitrarily set α_{33} =1, an innocuous specification because multiplying the α_{ij} parameters and dividing the γ_{ij} parameters by any positive constant leaves the structural equations unchanged.

With the parameter α_{33} set to one, attempts were made to estimate the other parameters however this was sometimes not successful. In some of the most restricted regressions, the γ parameters could be estimated however for some of the richer specifications, it was found that the set of γ parameter would uniformly converge to zero. This convergence increased the number of iterations and squeeze steps required before the TSP program would converge to a solution and was often unsuccessful because it would exceeded specified iteration or squeeze step limits.

Consider now the functional forms of μ and λ as the parameters γ_{ij} converge to zero. In this case, μ converges to min[B*,0]/A* and λ converges to max[B*,0]/C* as the discriminant converges to B^{*2}. This creates linear segments to the structural equations in μ and λ, pivoting around the β parameters. Because it is arguably more appealing that the structural equations should be "smooth" in μ and λ, because the likelihood changed little as the γ parameters converged to zero and, in particular, because of the difficulties of obtaining successful convergence as the γ_{ij}'s approached zero, γ_{11} was sometimes set as a constant.

Other specifications that aided estimation were holding parameters γ_{11}, γ_{22} and γ_{33} to be non-negative. It can be seen from the structural equations that these parameters are squared. Consequently, it is innocuous for the fit of the regression to use the positive values of these diagonal γ_{ii} parameters or truncate it at zero if the likelihood function is decreasing in this parameter.[17] Without this specification, the parameter γ_{ii} often seemed to oscillate explosively around zero for cases where likelihood was maximized at γ_{ii}=0. This truncation was also applied to the α_{ij} parameters to ensure non-negativity. Despite these aids to estimation, we found that in no case were we able to estimate parameters α_{ijj} so these were arbitrarily left at one. Thus, in the results we report below, some of the parameters are unrestricted while others are bound by our convexity restrictions or held as constants. Furthermore, the set of parameters that were binding differed depended on the regression performed. Nonetheless, the parameter estimates we report here are

[17] This was specified in TSP as $\gamma_{ii} = (\varphi_{ii} > 0)\varphi_{ii}$ where φ_{ii} was the parameter actually submitted into the regression. Within the parenthesis is a logical function that takes the value 1 if true and 0 if false.

convergent and robust in the sense of being independent of the many starting values we tried. We describe these regressions next.

For each of the three models: fixed effect, perfect foresight, and stochastic models; we performed four regressions: the basic, demographic, time-separable and general regressions. The log likelihoods of these 12 regressions and the number of free and constrained parameters are summarized by the 4x3 cells of table 2.

[TABLE 2 ABOUT HERE]

In each of the 12 cells of table 2, the top row gives the log likelihood of the corresponding model and regression. The second row of each cell gives two numbers: the total number of parameters estimated *within* the parenthesis and the number of free parameters estimated *outside* the parenthesis. The difference between the numbers inside and outside the parenthesis is the number of α_{ij} and γ_{ij} parameters constrained at zero due to convexity restrictions or held as a constant. The perfect foresight model involves one additional parameter over the fixed effect model, the time discount parameter δ. The stochastic model involves 5 additional parameters over the fixed effects model, the parameters δ, θ_w, θ_r, θ_y, and θ_a. While the perfect foresight model is not nested in the fixed effect model, the likelihood result suggests that it is a somewhat better fit than the fixed effects model. The estimate for the time discount factor parameter ranged from 0.61 to 0.91 in the four regressions with a value of 0.84 for the general regression.

The fixed effect model is nested in the stochastic model with θ_w, θ_r, θ_y, and θ_a all set to zero and δ set to one. It is clear on the basis of the likelihood ratio test that the hypothesis that these parameters are equal to zero is easily rejected in every regression. For this reason, and because the stochastic model allows for the determination of long-term and intertemporal effects we discuss later, we focus remaining discussion on the demographic, time separable and general regression of the stochastic model.

Table 3 reports parameter estimates for the demographic, time separable and general regressions for the stochastic model. Blank cells represent parameters not part of the model while cells with a value marked by an asterisk denote a parameter held at a constant, either because of a convexity restriction or to aid estimation as described above. The only case of the latter is in the stochastic model is parameter γ_{11} which was held in the general regression at the same value it was in the time-separable regression. We found that the likelihood was flat in this parameter but γ_{11} tended to drift towards zero, together with other γ parameters without a convergent solution.

Recall that we shall call the direct impact of sub-functions A, B and C, the "A-effect," the "B-effect" and the "C-effect" respectively. As discussed above, this captures the preferences of the infinitely rich, those in the middle and the infinitely poor.

One can see from parameter α_{11} in the demographic regressions that the marginal propensity to spend additional wealth on consumption is 1.8% for the infinitely rich. The marginal propensity to reduce labor earnings captured by parameter α_{22} in the demographic regression is 0.4%. In the time separable regression, these effects are captured as a cross price effect by parameter α_{12} and are economically small but statistically significant. In contrast, in the general regression, all estimated α parameters were bound by our convexity constraint suggesting that the propensity to consume or reduce labor earnings is zero for the infinitely rich.

The above A effect can be understood in the light of changes to the β parameters across the regressions. The parameters capturing the intercepts and pivot points of consumption, labor and wealth are, respectively, β_{110}, β_{120}, and β_{130} for male headed households. The corresponding parameters for female headed households are β_{210}, β_{220}, and β_{230}, respectively. In both the demographic and time-separable regressions, the B effect centers consumption at around \$26,000-\$31,000 for males and \$29,000-\$35,000 for females. In the general regression, the consumption pivot point shifts substantially, to \$87,000 for males and \$70,000 for females.

The change in the pivot points across regressions also occurs for labor and wealth. The demographic and time-separable regressions find an intercept of 1,730-1,820 and 1,180-1,320 hours per annum for male and female labor supply respectively while in the general regression the corresponding figures are -20 and 90 hours per annum respectively. It seems that there is also a change in the pivot points for wealth across the regressions however this is measured with considerably less precision that consumption or labor. The standard errors on the wealth intercepts are around an order of magnitude greater than the standard errors on consumption which are both measured in dollars. Nonetheless, it seems wealth pivots around -\$11,500 for males and \$51,000-\$58,000 for females in the demographic and time-separable regression which shifts substantially to $580,000 and $490,000 for males and females respectively in the general regression.

The demographic β parameters which conditions on age, age squared, age cubed and number of dependents estimated in all regressions except the basic regression show consistency across the regressions. Recall, the age variable we use is reported age of head minus 45 years in order to center the regression around prime aged heads. A comparison of parameters β_{120} and β_{220} show that- conditional on marginal utility, wages and interest rates- males at 45 work longer hours than females of the same age. However, parameters β_{121} and β_{221} show that males work 35 hours less per annum in the following year compared to females who work 18 hours per annum less the following year. The parameters β_{122} and β_{222} show that the decline in work hours past 45 years of ages is occurring at an increasing rate for both males and females. A comparison of parameters β_{130} and β_{230} show that females at 45 desire more wealth than males of the same age. Parameters β_{131} and β_{231} show that males desire an increase of $3,100-$4,400 in wealth in the following year after age 45 compared to females who desire an increase of $2,100-$2,800. The parameters β_{132} and β_{232} show that the desire to increase wealth is increasing past 45 years of ages for both males and females.

Among the parameters that capture effect of dependents on labor and wealth for males and females, only one, β_{224}, is consistently statistically significant. This estimate suggests females work around 45 hours more per annum for each dependent she has.

Although not statistically significant, the estimates are suggestive that males and females differ in their savings in response to the number of dependents. Each dependent reduces the wealth of males by around \$15,000-\$22,000 while it seems females save or provide an additional \$0-\$6,200 for each dependent. These results must be tempered in the light of the standard errors associated with the estimate.

[TABLE 3 ABOUT HERE]

The evaluation of the C effect is comparatively more difficult than it is for the A and B effects. The reason for this is that the γ parameters multiply prices. The structural equations of the basic regression (13) illustrate this. For illustrative purposes, let p = 1, w = 4.1 and r = 0.93. For the demographic regression using parameters $\gamma_{11}, \gamma_{22}, \gamma_{33}$, a one unit increase in λ decreases consumption by \$13,000 and assets by \$25,000 and increases work hours by 4,150 hours annually.

Table 4 gives quantitative indicators of fit for the system of 6 equations for the demographic, time separable and general regressions for the stochastic model. The R-square statistic are for the labor earnings equation $w_t h_t = w_t \pi_w(t) + \varepsilon_t$ equation for t = 1984 to 1988 and wealth equation $r_{1988} W_{1988} = r_{1988} \pi_r(1988) + \varepsilon_{1988}$. Looking across the regressions, relaxing consumption-labor additivity can improve the fit of the labor supply equations although the fit of the wealth equation falls. Despite this, it can be seen from the likelihoods and from table 3 that the two additional parameters, α_{12} and γ_{12}, are significant. By relaxing time separability, we have a comparatively larger increase in the likelihood which is mainly attributable to the improved fit of the wealth equation.

[TABLE 4 ABOUT HERE]

We show the qualitative fit of our consumption, labor and wealth equations for the general regression in figures 1.a, 1.b and 1.c respectively. In these figures, we plot actual values on predicted values. If our model predicted perfectly, all points would lie on a 45 degree line. We see here that the model fits fairly well without obvious systematic errors. There appears to be some heteroskedasticity for our consumption equation although this doesn't appear as strong in our labor earnings or wealth equation. Note however that the consumption equation was not part of our system of estimating equations. One can also see the truncation of actual wealth we used. Our wealth equation gave negative predictions for a small portion of our sample.

[FIGURE 1.a ABOUT HERE]

[FIGURE 1.b ABOUT HERE]

[FIGURE 1.c ABOUT HERE]

Our final analysis of this model looks at the impact of restrictions that are commonly employed in the literature and their impact on various estimated elasticities. For the structural equation using say end of period assets as an example,

$-A^* = \pi_r(p, w, r, f\mu)$, we define Frisch elasticity as the derivative of the log of asset with respect to the log of any of the arguments and denote this ε^F_{Aj}, j={p,w,r, μ }. The first item in the subscript denotes the choice variable and is c for consumption and h for labor hours. We define short term Marshallian elasticities by recognizing the cross-sectional dependence of μ_i on prices and beginning wealth. Again using end of period assets as an example, $-A^* = \pi_r(p, w, r, f\mu(p, w, r, W))$, we define short term Marshallian elasticity of end of period asset with respect to j, $\varepsilon^M_{Aj} = \varepsilon^F_{Aj} + \varepsilon^F_{A\mu}\varepsilon^{\mu}_{\mu j}$ where ε_{Aj} denotes the elasticity with respect to j={p,w,r,W} and superscript M and F denote Marshallian and Frisch elasticities respectively and superscript μ denotes the elasticity of μ with respect to any of its arguments. It should be understood that the Frisch elasticity with respect to beginning wealth is zero since Frisch elasticities condition on μ, not wealth.

We report Frisch and Marshallian elasticities for rich which we define as those with beginning wealth and exogenous income, W = 100,000 and poor households which we define as those with W = 50,000. Other prices, held at sample means, are p = 1, w = 4.1 and r = 0.93 and we assume the individual is 45 years of age with no dependents. Table 5 reports predicted consumption, labor and wealth of our rich and poor householders which should be read in conjunction with table 1 which reports mean values of the sample for interpretation.[18]

[18] The consumption figure in table 1 is actual 5 year composite consumption (see equation (16)) whereas the consumption figure here is predicted one year consumption. Otherwise, actual figures of table 1 and the predicted figures of table 5 are comparable. Additionally, we are not attempting to predict the means of

[TABLE 5 ABOUT HERE]

Table 5 can be evaluated in many respects. In comparing between rich and poor households we see that the main impact of greater wealth is the perpetuation of high wealth holdings. Rich households consume more than poor households although the increment is proportionately less than it is for wealth. Note also that rich work less than poor by about 80-100 hours annually. Additionally, although we saw that there were substantial changes in the A, B and C effects discussed above between the restricted regressions and the most general regression, these changes do not manifest themselves to a great extent in predicted quantities. End of period asset, consumption and work hours are comparable across the regressions.

In tables 6 and 7, we compare the Frisch and Marshallian elasticities respectively for rich and poor households. We see across the regressions of table 6 the effect of excluding certain cross-prices in the estimation of elasticities and how, in the general model, the full set of elasticities can be estimated. Because of the interconnection between Frisch and Marshallian elasticities given above, greater generality affect all estimates.

[TABLE 6 ABOUT HERE]

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table 1 but rather to give readers a sense of where our hypothetical rich and poor household is situated with respect to the actual data.

[TABLE 7 ABOUT HERE]

We draw attention to a few elasticities to illustrate the model and the effect of the restrictions. Consider the Frisch elasticity of consumption with respect to interest factor. Note that because $r_t = p_{t+1}/(1+i_t)$, a *one percentage point increase* in i_t will result in an approximate one percent *decrease* in r_t. With time separability, the interest factor does not exert an independent effect conditional on μ as seen in the Frisch elasticities of table 6. When this channel is allowed for in our general regression, we see a statistically significant and considerable effect which becomes stronger for poorer households. Going now to the same cells of table 7 which report Marshallian elasticities, we see with the time separable regression that a one percentage point increase in interest rates lead to a 0.046% and 0.152% decrease in consumption for rich and poor households respectively. On the other hand, when this effect is estimated with the general regression, one finds a substantial change with a one percentage point increase in interest rates leading to a 0.255% and 0.581% decrease in consumption for rich and poor households respectively. Other patterns that are noteworthy are that the positively sloped labor supply curve becomes appreciably more inelastic when estimated with the general regression and that the effect of increased wealth on labor supply differentially affects rich and poor households.

Our stochastic model has one other feature which is noteworthy: the ability to distinguish between household variation in μ_i arising from cross sectional variation and within household across time variation in $\mu_{i,t}$ arising from innovations in wages, interest rates or wealth. From above, μ_i depends on prices, wages, interest factors, exogenous income and starting wealth and its *variation* among households is a function of cross-sectional household variation of wages, interest factors, exogenous income and starting wealth. Consequently, an increase of the wage in say 1985 is combined with wages in other years, interest factors, exogenous income and so forth before it results in an increase in μ_i.

However, we can also compute a corresponding elasticity of $\mu_{i,t}$ with respect to wages, interest factors or assets allowing for the impact of function f. Given,

$$\mu_{i,t} = \frac{\delta^{(1988-t)} p_t}{p_{1988}} \exp(\theta_w (\tilde{w}_{i,t} - \tilde{w}_i) + \theta_r (\tilde{r}_{i,t} - \tilde{r}_i) + \theta_y (\tilde{y}_{i,t} - \tilde{y}_i) + \theta_a (A_{i,t} - A_i))\mu_i \text{, the}$$

elasticity of $\mu_{i,t}$ with respect to its argument adds a component $\varepsilon^f_{fj} = j\theta_j$ where index j= w, r, y and A and here denotes real wage, real interest factor, real exogenous income and asset respectively. The homogeneity of function f implies that the sum of elasticities with respect to p, w, r, y and W add to zero which allows for the recovery of the elasticity with respect to p. Table 8 reports the elasticity of μ_i with respect to wages, interest factors and wealth for rich and poor in the first and second panel. In the third panel, the elasticity of function f with respect to real wages, real interest factors, real exogenous income and assets is reported.

[TABLE 8 ABOUT HERE]

A remarkable parallel exists between the elasticity of μ_i for rich households and the elasticity f with respect to increases in assets. The elasticity of μ_i with respect to wealth for rich households is 0.915 in the demographic regression while the corresponding elasticity with respect to f is 0.401. In the general regression however, both elasticities seem to decline towards a similar magnitude, 0.178 and 0.179 respectively. This result is remarkable because these elasticities are derived from different functional forms and different aspects of the data. The cross sectional variation of wealth for the sample gives us our estimate of this elasticity which is determined collectively by α, β and γ parameters. On the other hand, *growth* in the level of individual wealth over a five year span give us our estimate of time series elasticity which is solely a function of the parameter θ_a. The consistency between these two estimates suggests this elasticity is estimated with some accuracy.

The elasticity of function f with respect to the interest factor across the regressions is rather interesting. Recall, the purpose of function f is to capture the evolution of marginal utility across time. Intertemporal optimization suggests the Euler equation, $\delta E_t(\lambda_{t+1}) = r_t \lambda_t / p_t$, where households equate the discounted expected future marginal utility of wealth saved with present marginal utility of wealth. Taking logs of this expression, it can be seen that the elasticity of μ_t with respect to r_t should be close to one.[19] The elasticity of function f with respect to the interest factor is estimated at 0.206 and 0.232 in the demographic and time separable regressions respectively. On the other hand, in the general regression, this elasticity is estimated at 1.084 and is not statistically different from one. Additionally, the general regression estimates this elasticity with greater precision than the demographic and time separable regression judged by the standard errors associated with this estimate across the regressions. Because the general regression allow for the interest factor to enter into the structural equations explaining consumption and labor supply, it appears that removing this mechanism in the restrictive regressions biases the estimate of how μ_t varies with interest factor and, incorrectly, suggests households are poor intertemporal optimizers.

The consistency of cross-sectional and time series elasticity estimates with respect to wealth contrasts notably with that of wages. The elasticity of μ with respect to wages is 0.072 and 0.112 for rich and poor household respectively in the demographic regression. On the other hand, the elasticity of f with respect to wages is 0.880 in the same regression. In the general regression, the elasticity of μ with respect to wages is 0.026 for rich and poor households while the elasticity of f with respect to wages is 0.168, a figure 6.5 times greater.

We account for this by the different construction of these elasticity estimates. Consider the case where a household has constant real interest factor $r_t/p_t = 1$, has a fixed

[19] An expectation of Y given X, E(Y|X), is a function of X, not a function of Y. Thus, time t marginal utility will be a function of the time t interest factor, price, the discount factor and anything else that is part of the information set of the household at time t. If r_t is not part of the information set of the household, then the elasticity will be exactly equal to one.

wage and works a fixed number of hours per annum over 5 years. In this simple case, an increase in wages in say 1984 has exactly the same impact as a similar increase in say 1989. Even in the more general case, the elasticity of μ with respect to wages combines many other variables into a single scalar measure. Although higher wages are likely to persist, no account of this is taken with our cross sectional measure of elasticity.

On the other hand, this restriction is not imposed in our time series estimate of elasticity. The impact of a positive wage shock on the price of marginal utility is likely to have a larger impact because this wage might be thought of a likely to persist for some time. One can imagine, for example, an individual with low wages for the first 4 years of our sample with an unexpected increase in wages in the final year of our sample. This individual may capitalize the value of higher future wages and adjust consumption, work hours and wealth targets by a larger amount than a similarly situated individual whose higher wages will fall to the lower level in subsequent years.

If such an interpretation is correct, we can now compute long term Marshallian elasticities. We define long term Marshallian elasticities as that which not only recognize the dependence of individual specific prices and beginning wealth on μ_i, but also allows for the time t perturbation captured by function f. Using end of period assets as the example once more, $-A^* = \pi_r(p, w, r, f(p, w, r, y, W)\mu(p, w, r, W))$, this elasticity is defined as $\varepsilon^{LM}_{Aj} = \varepsilon^{F}_{Aj} + \varepsilon^{F}_{A\mu}(\varepsilon^{f}_{fj} + \varepsilon^{\mu}_{\mu j})$, where the superscript LM is used. The new term with the superscript f denotes the elasticity of function f with respect to j={p,w,r,y}. The elasticity of function f is calculated with respect to y rather than W because y is a flow variable that is likely to persist into the future will W is an initial stock. Such defined long term Marshallian elasticities are reported in table 9.

The most notable feature of these estimates when compared to those of table 7 giving short term Marshallian elasticities is the effect of wages. For example, the short term elasticity of a rich household to a one percentage increase in wages on consumption, labor supply and wealth is 0.103%, 0.023% and 0.057% respectively but when this elasticity is evaluated over the long term, the corresponding change is 0.461%, -0.146% and 1.101%. The labor supply curve to long term wages is now backward bending. Looking across regressions, the slopes of the more restrictive functional forms understate the extent of the backward bend in labor.

CONCLUSION

Our paper sought to examine if duality techniques and greater generality can be profitably employed in the modeling of dynamic household choice. Our model treats intertemporal choice as a three good problem with choice variables consumption, labor supply and savings subject to a budget constraint- a treatment very similar to techniques used by demand modelers. In doing so, the invention of a new functional form was required which allows for the inversion of the budget constraint to determine an explicit expression for the unobserved marginal utility of income. This is, arguably, the most substantive contribution of this paper.

In addition to meeting this necessary requirement, our functional form has appealing properties. Our functional form is globally regular, rank 3 and is derived from a Laurent series approximation rather than the Taylor series approximation often used in flexible functional forms. Empirically, it can be seen that the model fits the data well.

With this new model, we tested two commonly maintained hypotheses and decisively rejected both consumption-labor additivity and time separability. These restrictions on the primal maximization problem amount- in the dual- to an imposition of zero cross-price Frisch elasticities which we find are restrictions that should be rejected. Removing these cross-price restrictions changes the entire set of estimated Frisch and Marshallian elasticities. Additionally, important intertemporal parameters such as the rate of time preference and the time series elasticity of the marginal utility of income with respect to changes in wage, interest factor and wealth are more precisely measured by our more general regression.

Our results cast doubt on contemporary elasticity estimates made using the more restrictive forms. For example, we find that the estimated decline of consumption with respect to interest rates increases is greatly understated (by a factor of around 5) by restrictive regressions. We find that a one percent increase in interest rates decrease consumption of rich and poor households by 0.255 and 0.581 percent respectively instead of 0.046% and 0.152%. The common presumption that labor supply is positively sloped was found only to apply to transitory wage increases. Should the wage increase be permanent or at least persistent, the wealth effect of higher wages causes labor supply to contract. May elasticities are affected by separability assumptions which are all detailed in the preceding tables.

However, more than the particulars of our findings, what we hope we offer to the profession is a fruitful approach that opens new areas in economics. To this end, we offer what we consider worthwhile extensions of our framework. Some of these are merely technical refinements but others have the potential to affect other fields of economics.

On a technical level, we estimated preferences for single headed households using what amounts to a single cross-section. Although the data was a panel, it was necessary to have beginning and end wealth to determine consumption. We do not have true “within” and “between" errors analogous to variance component models. To do this, a third wealth supplement is required. This would allow analysis of household behavior across time and in cross-section. This was not done in this paper because, at the time of writing, final release PSID data for the next wealth supplement in 1994 was unavailable. The use of *early release* data would have allowed for the analysis to include wealth from the 1994, 1999 and 2001 waves of the PSID building a true panel structure for the data. We did not attempt this however because we relied on numerous constructed variables unavailable in the early release files, for example federal taxes.

Another important development would be extending this to couples. The number of married and joint households is 3 times larger that single headed households which allows for a better analysis simply from having more data alone. The household intertemporal problem can be conceived of as a four good problem: how much to consume, how much to save, and the labor supply of the head and the spouse as separate choices. It would be interesting to allow for the interaction between head and spouse labor supply.

Another extension is to model the demand for particular classes of assets. We aggregated all forms of wealth and derived a composite return of this wealth. However, given that that the PSID has the individual return of many assets, it would seem possible to treat these as distinct goods, each with its own price. This would then allow the analysis of substitutability or complementarity of the different assets. Another extension or refinement would be to further generalize the functional form. For example, the literature on precautionary savings suggests that income volatility increases the demand for wealth. This analysis can be captured in our framework by incorporating income volatility measures in our sub-function C since it is the poor that are most likely to be intertemporally constrained.

In general, what was surprising to us was that every generalization we attempted- going from the basic, to the demographic, time separable and general regressions, and incorporation of θ parameters one at a time in function f- proved to be statistically significant. This suggests that the search for even greater generality and improved fit has not been exhausted. While this is true, it is also true that estimation at times proved to be a substantial challenge. Yet another refinement would be to improve the efficiency of our estimation procedure.

Because of the central importance of household choice in much of economics, our results have wide-ranging implications for other fields in economics and for public policy. Our results should be incorporated into business cycle theories where modeling the dynamic behavior of households is of central importance. Another application is in the field of social welfare functions. Our model identifies the marginal utility of income which occupies a central place in analyses that consider the redistribution of income. A further application is in the area of tax incidence. A fundamental policy choice is the balance of taxes on consumption (sales and value added taxes), labor earnings (social security, payroll and income taxes) and wealth (corporate income, property and wealth taxes) and our model informs on each of these elasticities. Another application is in general equilibrium models where the flows to and from firms are matched with the flows from and to households. The supply of consumption by firms to household and the supply of labor by households to firms are obvious, but it would also seem possible that the wealth demand of households could be translated into the capital requirement of firms.

We offer our procedure in the hope economists find this fertile ground.

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Table 1. Summary statistics of selected sample in 1988 by gender.

	Male headed households (n=92)				Female headed households (n=433)			
Variable	mean	std. dev.	minimum	maximum	mean	std. dev.	minimum	maximum
Age of Head	56.5	17.6	27	90	63.1	16.2	25	97
# of dependents	1.3	0.75	1	6	1.3	0.81	0	8
Consumption ($)	80,178	45,055	10,802	209,461	61,130	39,823	6,397	270,458
Wealth ($)	73,950	79,257	1,500	439,000	69,587	91,392	1,200	825,000
Interest return (i)	0.071	0.086	-0.184	0.335	0.076	0.093	-0.199	0.338
Exogenous income ($y)	1,998	8,643	-21,918	34,320	5,172	15,116	-10,399	255,953
Head work hours *	1,907	586	120	2,880	1,666	611	10	2,975
Labor Earnings ($) *	16,144	13,243	441	66,861	10,243	8,120	65	42,947

* summary statistics reported only for the 61 males and 203 females working in 1989.

Table 2. Log likelihood fit of various regressions and models

	Fixed Effect Model	Perfect Foresight Model	Stochastic Model
Basic Regression	-29,982.8 9(10)	-29,947.9 10(11)	-29,592.8 16(16)
Demographic Regression	-29,550.6 24(26)	-29,543.8 26(27)	-29,353.4 32(32)
Time Separable Regression	-29,539.2 25(28)	-29,533.5 26(29)	-29,342.0 32(34)
General Regression	-29,533.0 26(32)	-29,528.4 26(33)	-29,264.8 31(38)

Table 3. Parameter estimates of the stochastic model across 3 regressions

	Demographic		Time Separable		General	
Parameter	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
α_{11}	0.017944	6.76E-03	0*		0*	
α_{12}			3.73E-03	8.29E-04	0*	
α_{13}					0*	
α_{22}	3.96E-03	6.58E-04	0*		0*	
α_{23}					0*	
β_{110}	-25796.7	1427.99	-31458.1	1269.78	-86885.9	5662.81
β_{210}	-28630	1327.27	-35339.3	1019.62	-69537.4	4004.72

β_{120}	1822.79	34.9589	1733.43	37.832	-22.9231	155.579
β_{121}	-34.0907	2.08425	-35.0078	2.18231	-28.4397	2.66114
β_{122}	-1.4697	0.13809	-1.61456	0.140394	-1.68049	0.189105
β_{123}	0.022245	5.69E-03	0.025722	5.74E-03	0.027636	6.76E-03
β_{124}	-3.72301	13.0213	-0.12819	12.5666	31.1976	13.8727
β_{220}	1315.08	34.8169	1176.44	40.9519	91.9538	101.084
β_{221}	-18.2354	2.11219	-17.6295	2.03109	-20.5572	2.02819
β_{222}	-0.83295	0.123286	-0.79751	0.121067	-0.67956	0.113292
β_{223}	0.012568	5.63E-03	8.88E-03	5.37E-03	0.011064	5.15E-03
β_{224}	44.4819	14.9879	44.9085	14.7947	45.8399	14.5682
β_{130}	11805.6	18653.2	11429.5	19354.9	-582342	49666.9
β_{131}	-4395.32	800.485	-3892.46	913.959	-3147.34	845.961
β_{132}	-191.183	51.6868	-218.434	56.6515	-352.551	61.8298
β_{133}	5.23984	1.52838	5.54467	1.69915	8.3224	1.59095
β_{134}	15065.4	20767.1	15083.3	15809.4	22120.5	12679.8
β_{230}	-50698	8616.17	-57797.3	11188.7	-489221	32123.8
β_{231}	-2148.1	454.897	-2307.77	482.224	-2783.27	731.24
β_{232}	-115.81	31.2484	-109.766	32.3231	-57.1472	47.8156
β_{233}	2.76845	0.681474	2.66995	0.695061	2.01035	0.935815
β_{234}	-6245.56	4445.54	-4916.96	4660.93	526.542	7388.93
γ_{11}	13080.1	1519.95	19990.9	2391.25	19990.9*	
γ_{12}			353.387	79.0996	-271.067	44.8451
γ_{13}					33243.6	8363.41
γ_{22}	1011.93	112.395	1083.37	121.909	529.301	62.8834
γ_{23}					85504.8	4974.84
γ_{33}	27019.2	2470.59	38976.8	4126.3	0*	
θ_w	0.214725	9.28E-03	0.165366	8.82E-03	0.040877	2.81E-03
θ_r	0.221297	0.385137	0.249454	0.290191	1.1661	0.081354
θ_y	1.04E-05	1.13E-06	1.04E-05	9.81E-07	1.00E-05	8.66E-07
θ_a	4.01E-06	1.29E-06	1.92E-06	9.84E-07	1.79E-06	2.42E-07
δ	0.950	0.012	0.954	0.010	0.985	0.003

0* indicates a binding convexity restriction on the parameter.

Table 4. Statistics of fit for labor earnings and wealth equations for stochastic model across demographic, time separable and general regressions.

Statistic of Fit	Demographic	Time Separable	General
R^2: labor 1984	0.881	0.881	0.885
R^2: labor 1985	0.891	0.890	0.894

R^2: labor 1986	0.918	0.917	0.914
R^2: labor 1987	0.924	0.926	0.921
R^2: labor 1988	0.932	0.932	0.926
R^2: wealth 1989	0.780	0.767	0.787
Log. Likelihood	-29353.4	-29342.0	-29264.8

Table 5. Predicted consumption, labor and wealth for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
Assets (1989 dollars)	87547.6	925.3	85920.7	1032.6	85489.6	1572.4
Consumption (1989 dollars)	25837.6	853.4	27383.0	945.5	27669.7	1428.6
Annual Work Hours	1770.0	25.3	1777.9	26.1	1750.0	40.1
	Poor Household					
Assets (1989 dollars)	36353.3	968.0	35371.6	1146.6	38100.7	1576.8
Consumption (1989 dollars)	23750.3	882.6	24812.5	1016.9	22389.5	1430.2
Annual Work Hours	1843.6	27.3	1880.0	33.4	1908.1	40.9

Table 6. Frisch elasticities for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^F_{cp}	-0.136	0.025	-0.142	0.025	-0.858	0.131
ε^F_{cw}			-0.012	0.003	0.048	0.010
ε^F_{cr}					-1.329	0.226
$\varepsilon^F_{c\mu}$	0.136	0.025	0.154	0.027	2.140	0.236
ε^F_{hp}			0.047	0.010	-0.184	0.040
ε^F_{hw}	0.078	0.011	0.063	0.008	0.049	0.008
ε^F_{hr}					1.148	0.083
$\varepsilon^F_{h\mu}$	-0.078	0.011	-0.110	0.015	-1.013	0.089
ε^F_{Ap}					-0.463	0.076
ε^F_{Aw}					-0.104	0.007
ε^F_{Ar}	-1.292	0.197	-1.452	0.212	-5.668	0.508
$\varepsilon^F_{A\mu}$	1.292	0.197	1.452	0.212	6.234	0.577
	Poor Household					

ε^{F}_{cp}	-0.171	0.027	-0.252	0.048	-1.156	0.188
ε^{F}_{cw}			-0.020	0.005	0.064	0.014
ε^{F}_{cr}					-1.789	0.301
$\varepsilon^{F}_{c\mu}$	0.171	0.027	0.272	0.052	2.881	0.327
ε^{F}_{hp}			0.064	0.013	-0.184	0.040
ε^{F}_{hw}	0.070	0.008	0.064	0.007	0.049	0.008
ε^{F}_{hr}					1.147	0.076
$\varepsilon^{F}_{h\mu}$	-0.070	0.008	-0.128	0.017	-1.012	0.082
ε^{F}_{Ap}					-1.131	0.183
ε^{F}_{Aw}					-0.253	0.017
ε^{F}_{Ar}	-1.991	0.399	-2.571	0.446	-13.770	1.241
$\varepsilon^{F}_{A\mu}$	1.991	0.399	2.571	0.446	15.154	1.405

Table 7. Marshallian elasticities for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^{M}_{cp}	-0.163	0.027	-0.171	0.028	-0.738	0.124
ε^{M}_{cw}	0.010	0.002	-0.002	0.003	0.103	0.011
ε^{M}_{cr}	0.030	0.018	0.046	0.018	0.255	0.108
ε^{M}_{cW}	0.124	0.022	0.128	0.026	0.381	0.024
ε^{M}_{hp}	0.016	0.002	0.068	0.011	-0.241	0.039
ε^{M}_{hw}	0.072	0.011	0.056	0.008	0.023	0.007
ε^{M}_{hr}	-0.017	0.010	-0.033	0.013	0.398	0.036
ε^{M}_{hW}	-0.071	0.009	-0.091	0.013	-0.180	0.008
ε^{M}_{Ap}	-0.264	0.015	-0.278	0.017	-0.113	0.044
ε^{M}_{Aw}	0.092	0.002	0.097	0.002	0.057	0.004
ε^{M}_{Ar}	-1.011	0.007	-1.019	0.007	-1.053	0.035
ε^{M}_{AW}	1.182	0.014	1.199	0.016	1.109	0.021
	Poor Household					
ε^{M}_{cp}	-0.218	0.035	-0.305	0.055	-0.956	0.174

ε^{M}_{cw}	0.019	0.005	0.006	0.006	0.139	0.016
ε^{M}_{cr}	0.080	0.017	0.152	0.028	0.581	0.155
ε^{M}_{cW}	0.119	0.031	0.147	0.037	0.236	0.019
ε^{M}_{hp}	0.019	0.003	0.089	0.014	-0.254	0.039
ε^{M}_{hw}	0.062	0.007	0.052	0.006	0.023	0.007
ε^{M}_{hr}	-0.033	0.009	-0.072	0.014	0.314	0.035
ε^{M}_{hW}	-0.049	0.008	-0.069	0.011	-0.083	0.003
ε^{M}_{Ap}	-0.545	0.050	-0.503	0.071	-0.084	0.108
ε^{M}_{Aw}	0.224	0.007	0.242	0.009	0.138	0.011
ε^{M}_{Ar}	-1.063	0.015	-1.131	0.024	-1.298	0.087
ε^{M}_{AW}	1.385	0.049	1.393	0.063	1.243	0.052

Table 8. Cross sectional μ and function f elasticities.

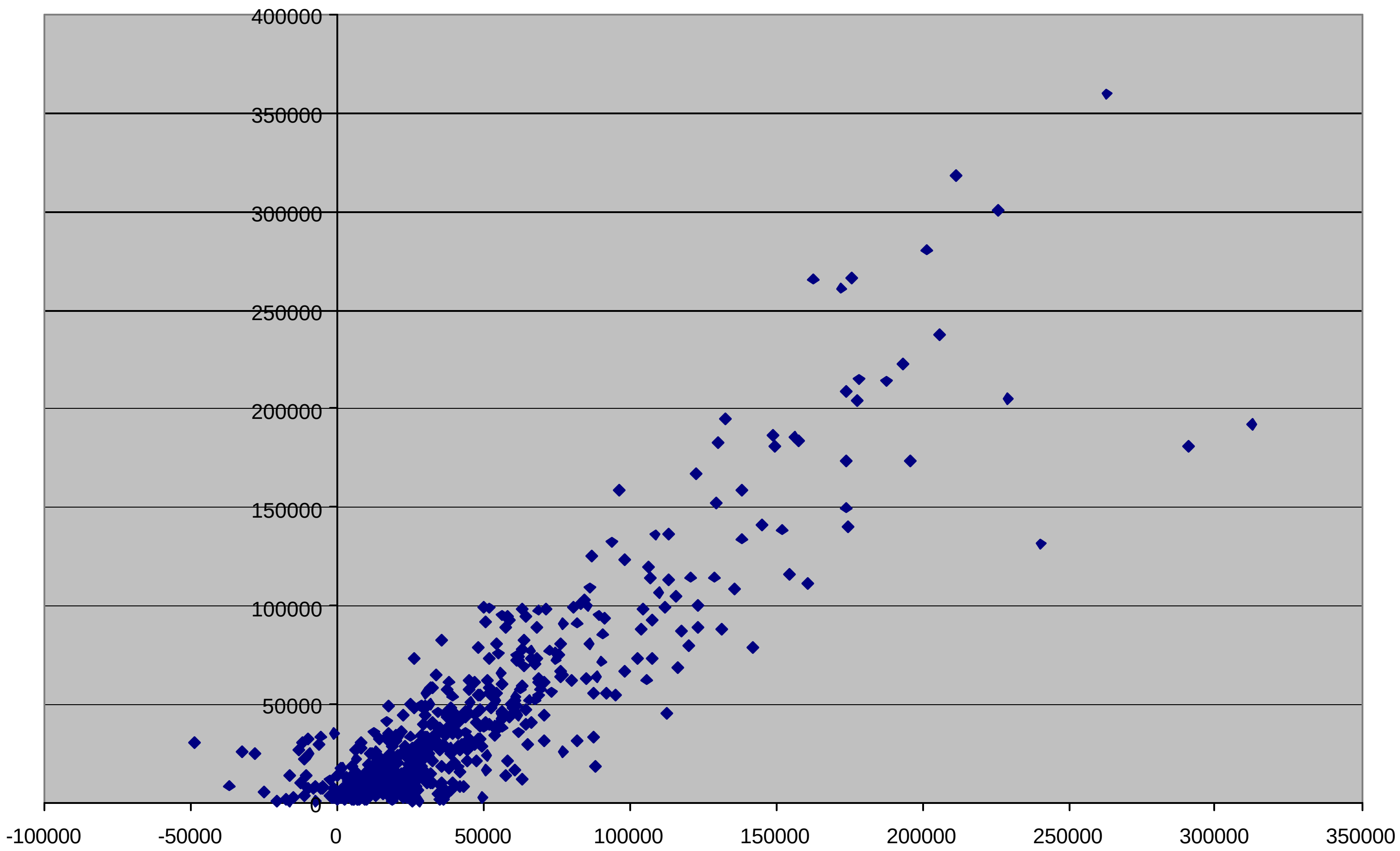
	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
$\varepsilon^{\mu}_{\mu w}$	-0.204	0.035	-0.191	0.030	0.056	0.006
$\varepsilon^{\mu}_{\mu r}$	0.072	0.011	0.067	0.009	0.026	0.001
$\varepsilon^{\mu}_{\mu A}$	0.218	0.114	0.298	0.098	0.740	0.014
	Poor Household					
$\varepsilon^{\mu}_{\mu w}$	-0.204	0.035	-0.191	0.030	0.056	0.006
$\varepsilon^{\mu}_{\mu r}$	0.072	0.011	0.067	0.009	0.026	0.001
$\varepsilon^{\mu}_{\mu A}$	0.218	0.114	0.298	0.098	0.740	0.014
	Elasticity of function f.					
ε^{f}_{fw}	0.880	0.038	0.678	0.036	0.168	0.012
ε^{f}_{fr}	0.206	0.358	0.232	0.270	1.084	0.076
ε^{f}_{fy}	1.041	0.113	1.036	0.098	1.002	0.087
ε^{f}_{fA}	0.401	0.129	0.192	0.098	0.179	0.024

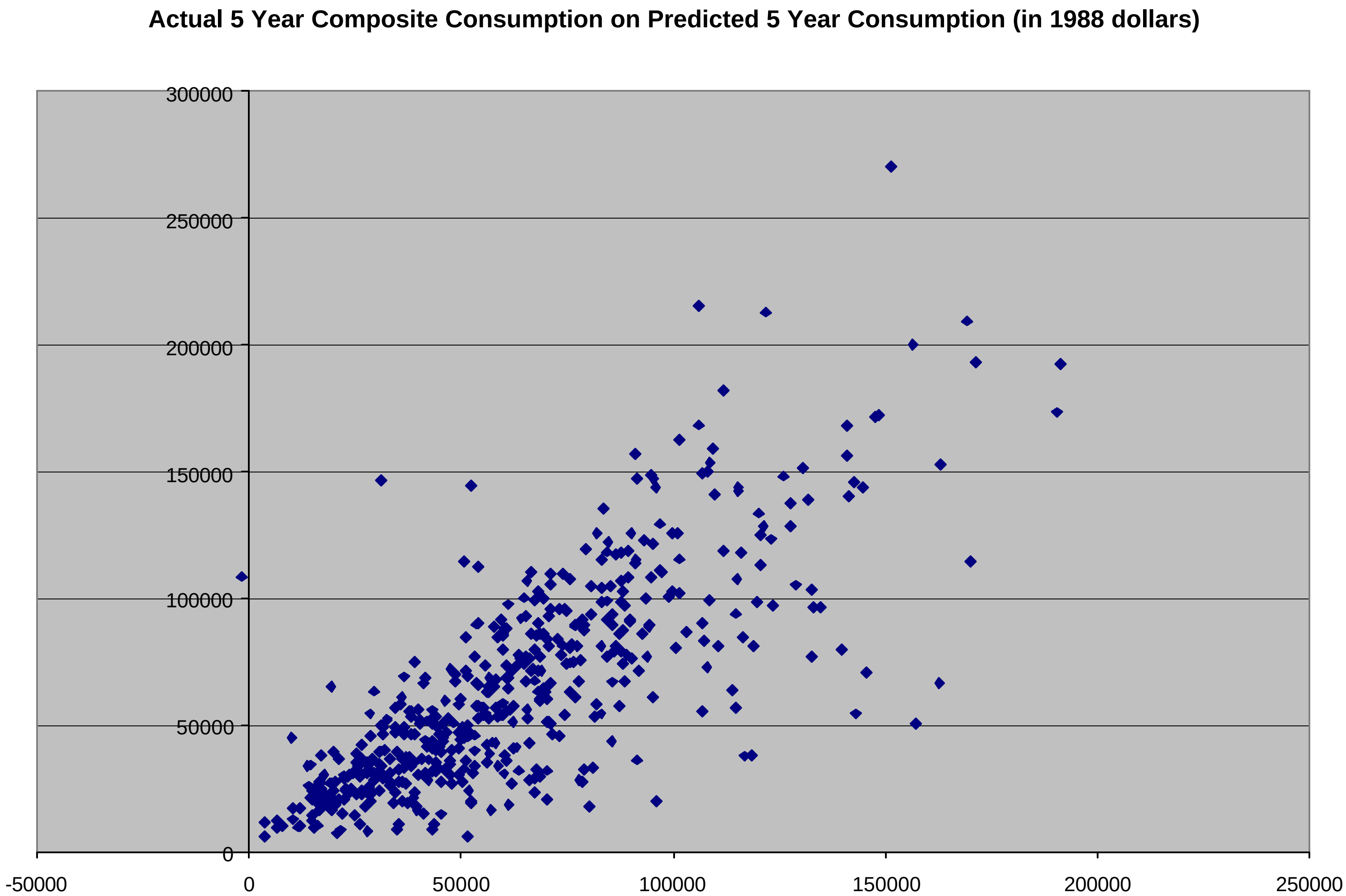
Table 9. Long term Marshallian Elasticities.

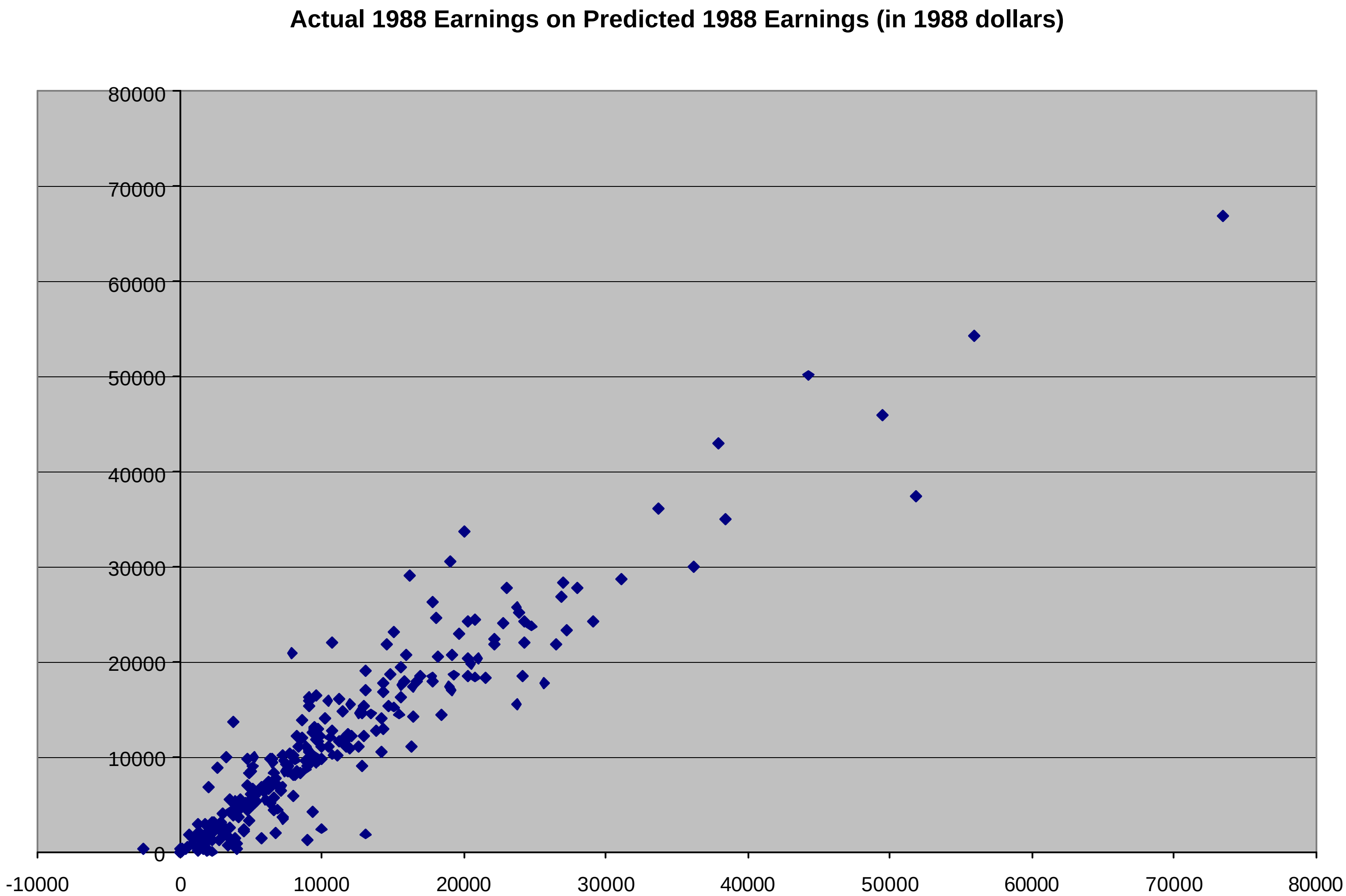
	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.

	Rich Household					
ε_{cp}^{LM}	-0.452	0.094	-0.472	0.079	-5.562	0.545
ε_{cw}^{LM}	0.129	0.023	0.102	0.016	0.461	0.037
ε_{cr}^{LM}	0.057	0.055	0.082	0.046	2.576	0.317
ε_{cW}^{LM}	0.265	0.043	0.287	0.048	2.524	0.230
ε_{hp}^{LM}	0.182	0.036	0.282	0.045	2.042	0.135
ε_{hw}^{LM}	0.004	0.004	-0.019	0.005	-0.146	0.007
ε_{hr}^{LM}	-0.033	0.031	-0.058	0.034	-0.701	0.102
ε_{hW}^{LM}	-0.152	0.017	-0.205	0.024	-1.195	0.059
ε_{Ap}^{LM}	-3.012	0.638	-3.103	0.527	-14.163	0.991
ε_{Aw}^{LM}	1.230	0.172	1.081	0.130	1.101	0.066
ε_{Ar}^{LM}	-0.745	0.466	-0.682	0.396	5.708	0.658
ε_{AW}^{LM}	2.527	0.236	2.704	0.206	7.353	0.420
	Poor Household					
ε_{cp}^{LM}	-0.492	0.088	-0.693	0.138	-6.006	0.653
ε_{cw}^{LM}	0.170	0.026	0.190	0.037	0.622	0.055
ε_{cr}^{LM}	0.115	0.062	0.215	0.080	3.705	0.460
ε_{cW}^{LM}	0.208	0.043	0.288	0.063	1.679	0.172
ε_{hp}^{LM}	0.132	0.026	0.271	0.048	1.520	0.110
ε_{hw}^{LM}	0.001	0.003	-0.034	0.008	-0.146	0.006
ε_{hr}^{LM}	-0.047	0.027	-0.101	0.039	-0.784	0.098
ε_{hW}^{LM}	-0.085	0.011	-0.135	0.018	-0.590	0.028
ε_{Ap}^{LM}	-3.745	0.984	-4.174	0.918	-26.647	2.129
ε_{Aw}^{LM}	1.977	0.345	1.985	0.274	2.678	0.188
ε_{Ar}^{LM}	-0.653	0.720	-0.535	0.704	15.136	1.618
ε_{AW}^{LM}	2.421	0.253	2.725	0.252	8.833	0.581

Actual 1989 Wealth on Predicted 1989 Wealth (in 1988 dollars)




Actual 5 Year Composite Consumption on Predicted 5 Year Consumption (in 1988 dollars)
300000
250000
200000
150000
100000
50000
0
-50000
0
50000
100000
150000
200000
250000


Actual 1988 Earnings on Predicted 1988 Earnings (in 1988 dollars)
80000
70000
60000
50000
40000
30000
20000
10000
0
-10000
0
10000
20000
30000
40000
50000
60000
70000
80000